# Math 208 Midterm I

April 23, 2025

### NAME:

## UW EMAIL:

## STUDENT ID NUMBER:

1	/10
2	/10
3	/10
4	/10
5	/10
6	/10
Bonus	/2
Total	/60

#### Instructions.<sup>1</sup>

- For each problem below give a carefully explained solution using the vocabulary and notation from class. A correct answer with no supporting work or explanation will receive a zero.
- Put a box around your final answers.
- If a solution involves a numerical answer, collect all terms and reduce all fractions. No decimal expansion is necessary.
- You are allowed a simple calculator and notesheet. Other notes, electronic devices, etc are not allowed. Take a few pencils from your pencil case out and put all other items away for the duration of the exam.
- All the questions can be solved using (at most) simple arithmetic. (If you find yourself doing complicated calculations, there might be an easier solution...)
- Raise your hand if you have any questions or spot a possible error.
- Please stay in your seat until all exams are collected and the class is dismissed.

Good luck!

<sup>&</sup>lt;sup>1</sup>Test code: 8742

(1) Let A be the augmented matrix

$$A = \left[ \begin{array}{rrrrr} 4 & -3 & 1 & 2 & 0 & | & 6 \\ 0 & 0 & -2 & 4 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & | & 1 \end{array} \right].$$

(a) (3 points) Write the system of linear equations that correspond with A.

(b) (3 points) Specify the leading variables in this system.

(c) (4 points) Is this system consistent? Justify your answer.

(2) (10 points) Find the general solution to the following system of linear equations by converting the system to an augmented matrix and finding its REDUCED row echelon form.

$$x_2 + 3x_4 = 7$$
  

$$x_1 + x_3 = 6$$
  

$$2x_1 + 4x_3 = 14$$

(3) (10 points) What linear equation(s) has solution set equal to the span of the following 3 vectors:

$$\begin{bmatrix} 0\\0\\0\\3 \end{bmatrix}, \begin{bmatrix} 2\\0\\0\\0 \end{bmatrix}, \begin{bmatrix} 0\\4\\2\\1 \end{bmatrix}$$

(4) Let 
$$A = \begin{bmatrix} 2 & 1 & | & -4 \\ -1 & 3 & | & 9 \end{bmatrix}$$
.

(a) (4 points) Starting with A, write out each step in the process of Gauss-Jordan elimination to find the reduced row echelon matrix equivalent to A, indicating the elementary row operation used.

(b) (3 points) For each matrix above, sketch a picture showing the corresponding lines intersecting in  $\mathbb{R}^2$  labeling the intersection of the lines with each other, the *x*-axis, and the *y*-axis.

(c) (3 points) How does the final sketch relate to the original augmented matrix in part (a)? (Hint: Use it to check your work.)

(5) (10 points) Find a vector that is NOT a linear combination of

$$\mathbf{u} = \begin{bmatrix} 3\\1\\-2\\1 \end{bmatrix} \text{ and } \mathbf{v} = \begin{bmatrix} -4\\2\\3\\3 \end{bmatrix}.$$

Justify your answer.

(6) For each of the following scenarios, write the system of linear equations IN STANDARD FORM you would need to solve in order to find the answer.

(a) (5 points) Nelson has p pennies, n nickels, and d dimes, for a total of 52 coins. A penny is worth 1 cent and weighs 2.5 grams. A nickel is worth 5 cents and weighs 5 grams. A dime is worth 10 cents and weighs 2.25 grams. In total, Nelson's coins are worth 242 cents and weigh 143 grams. How many of each type of coin does Nelson have?

(b) (5 points) Sara's YouTube video on Gauss Elimination received u thumbsup-votes and d thumbs-down-votes. The post received 35 more down-votes than up-votes; in fact, only 40% of the total votes received (up-votes and down-votes combined) were up-votes. How many up-votes and down-votes did the video receive? Gemini, CoPilot, Llama, etc for helping you to learn linear algebra?

If so, how do these systems help you?

If not, what are the reasons you are not using them?